Iowa Curve Average Life Discrepancies

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Most Iowa Curve listings are presumed to be scaled to a 100-year life; however, this is not always the case for many published Iowa curves. While the differences are small, less than one year in most cases, this discrepancy can result in a material margin of error in the calculation of the remaining lives. The magnitude of the error depends on the remaining life algorithm used.

There are many reasons these discrepancies exist, such as rounding errors, copying errors, scaling, etc. For instance, the initial publication, Bulletin 125, presented the percent surviving in 5% increments (i.e., age as a percent of average life). This equates to 5 years for a 100-year life. Restating the Iowa Curves to the more commonly found 1% curve listing, likely also contributed to this problem.

Deviations in the average life away from 100-years distorts the age increment and hence distorts the estimate of the remaining life. If the average life is not 100-years than the 1% age increment does not correspond to 1 year; hence, the sum of the percent survivors less ½ year (i.e., half the interval of integration[1]) will not equal the true average life. To the extent the average life of the Iowa curves does not equal 100, any estimate of the remaining life will be off accordingly.

We can resolve this problem, or at least greatly mitigate the margin of error, by accurately estimating the average life of each Iowa curve. This is accomplished through an iteration process, whereby with each iteration we refine the interval of integration (the period of time between successive percent survivors). I recommend a 3-pass iteration as follows:

1. Assume an Average Life of exactly 100 years.
    a. The first 1% interval of integration (I_1) is assumed to be 1 year (AveLife/100 or 100/100)
2. Compute the Average Life using I_1
    a. $AveLife_2 = \frac{\sum_{n=0}^{\infty} PS_n}{PS_0} - \frac{I_1}{2}$
        i. Note, for many published Iowa curves, the life will not exactly equal 100.
    b. Recompute the interval of integration: $I_2 = AveLife_2/100$
3. Recompute the Average Life:
    a. $AveLife_3 = \frac{\sum_{n=0}^{\infty} PS_n}{PS_0} - \frac{I_2}{2}$

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[1] The average life of a survivor curve is the area under the curve divided by the starting point. We estimate the area using the trapezoidal rule, which, for a complete survivor curve, states that the area approximately equals the sum of the percent survivors less one-half the **interval of integration** (i.e., the period of time between each percent surviving), not ½ year.

At this point, we have a reliable and accurate estimate of the average life of the Iowa curve from which to calculate the integral of integration, I. We only need to perform this 3-pass calculation of the average life once for each Iowa curve. My investigation suggest that the average life computed in pass 3 is accurate to at least 5 decimal places (when using the percent survivors with 5 significant digits). With this added level of accuracy, the remaining life calculations will yield more accurate results.

With a more precise estimate of the average life, the original survivor curve can now be accurately scaled to any projection life (PLife). Note, the time-period between each 1% age increment is not 1 year, but rather the refined average life divided by 100 for the original curve. For the scaled curve, the scaled age increment is simply the PLife divided by 100.

For publication, BCRI scaled the original Iowa curves to exactly a 100.00 year average life. For some Iowa curves, this results in a slight difference between the original Iowa curve percent surviving and BCRI's published version. When scaled to exactly 100-years, the age increment is exactly 1-year and one-half the **interval of integration** simply equals 0.5*PLife/100 or simply PLife/200. As such, computing the remaining life for any age is simpler and more accurate.

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